PhD Project

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March 25, 2024

The goal of this course has been to prepare you to engage with current research on dependent type theory, so your final project is to read and summarize a research paper on this topic. You may choose to do this in one of two ways:

1. Give a 30–45 minute "chalk talk" on a research paper.
2. Write a 5–10 page paper summarizing the results and ideas of the paper.

In both cases, you should target fellow Ph.D. students in this course, *i.e.,* at someone who understands the material presented thus far in the course. We encourage students in the course to attend the talks of those who choose to give presentations.

The presentation or summary should focus on *what* the goals and contributions of the paper are, *why* these are the goals, and *how* they are achieved. Spend time identifying the broader context in which the paper was written, including briefly summarizing some earlier papers as necessary. Include some substantive technical details but don't just paraphrase large blocks of text. Standard academic writing principles apply and try to give proper attribution for ideas.

Please let Daniel know via email as soon as possible which paper you intend to study and which method of presentation you prefer. We will announce a schedule for paper presentations once we have the relevant information.

Suggested papers

Here are some suggestions for suitable papers sorted roughly by topic. Particularly technical papers are marked with a ⋆. Feel free to propose a different paper that is *about* (not just *uses*) full-spectrum dependent type theory and its syntax, semantics, or implementation.

Even if you choose a paper listed below, we strongly suggest that you skim it before committing, and **chat with Daniel** about possible references to read first or sections you can skip. Note that longer papers are not necessarily harder!

Categorical semantics of type theory

- Martin Hofmann. "Syntax and Semantics of Dependent Types". In: *Semantics and Logics of Computation*. Ed. by Andrew M. Pitts and P. Dybjer. Publications of the Newton Institute. Cambridge University Press, 1997, pp. 79–130. DOI: 10.1017/CBO9780511526619.004
- Pierre-Louis Curien, Richard Garner, and Martin Hofmann. "Revisiting the categorical interpretation of dependent type theory". In: *Theoretical Computer Science* 546 (2014). Models of Interaction: Essays in Honour of Glynn Winskel, pp. 99–119. ISSN: 0304-3975. DOI: 10.1016/j.tcs.2014.03.003
- Steve Awodey. "Natural models of homotopy type theory". In: *Mathematical Structures in Computer Science* 28.2 (2018), pp. 241–286. DOI: 10.1017/S0960129516000268
- Simon Castellan, Pierre Clairambault, and Peter Dybjer. "Categories with Families: Unityped, Simply Typed, and Dependently Typed". In: *Joachim Lambek: The Interplay of Mathematics, Logic, and Linguistics*. Ed. by Claudia Casadio and Philip J. Scott. Cham: Springer International Publishing, 2021, pp. 135–180. ISBN: 978-3-030-66545-6. DOI: 10.1007/978-3-030-66545-6_5
- ⋆ Ian Orton and Andrew M. Pitts. "Axioms for Modelling Cubical Type Theory in a Topos". In: *25th EACSL Annual Conference on Computer Science Logic (CSL 2016)*. Ed. by Jean-Marc Talbot and Laurent Regnier. Vol. 62. Leibniz International Proceedings in Informatics (LIPIcs). Dagstuhl, Germany: Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, 2016, 24:1–24:19. ISBN: 978-3-95977-022-4. DOI: 10.4230/LIPIcs.CSL.2016.24

Logical frameworks

- Robert Harper, Furio Honsell, and Gordon Plotkin. "A framework for defining logics". In: *J. ACM* 40.1 (Jan. 1993), pp. 143–184. ISSN: 0004-5411. DOI: 10.1145/138027.138060
- Thorsten Altenkirch and Ambrus Kaposi. "Type theory in type theory using quotient inductive types". In: *Proceedings of the 43rd Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*. POPL 2016. New

York, NY, USA: ACM, 2016, pp. 18–29. ISBN: 9781450335492. DOI: 10.1145/2837614.2837638

- Ambrus Kaposi, András Kovács, and Thorsten Altenkirch. "Constructing quotient inductive-inductive types". In: *Proceedings of the ACM on Programming Languages* 3.POPL (Jan. 2019), 2:1–2:24. DOI: 10.1145/3290315

★ Taichi Uemura. "A general framework for the semantics of type theory". In: *Mathematical Structures in Computer Science* 33.3 (2023), pp. 134–179. DOI: 10.1017/S0960129523000208

Observational type theories

- Thorsten Altenkirch, Conor McBride, and Wouter Swierstra. "Observational Equality, Now!" In: *Proceedings of the 2007 Workshop on Programming Languages Meets Program Verification*. PLPV '07. New York, NY, USA: ACM, 2007, pp. 57–68. ISBN: 978-1-59593-677-6. DOI: 10.1145/1292597.1292608

- Jonathan Sterling, Carlo Angiuli, and Daniel Gratzer. "A Cubical Language for Bishop Sets". In: *Logical Methods in Computer Science* 18 (1 Mar. 2022). DOI: 10.46298/lmcs-18(1:43)2022

- Loïc Pujet and Nicolas Tabareau. "Observational Equality: Now for Good". In: *Proceedings of the ACM on Programming Languages* 6.POPL (Jan. 2022). DOI: 10.1145/3498693

- Loïc Pujet and Nicolas Tabareau. "Impredicative Observational Equality". In: *Proceedings of the ACM on Programming Languages* 7.POPL (Jan. 2023). DOI: 10.1145/3571739

★ Thorsten Altenkirch, Yorgo Chamoun, Ambrus Kaposi, and Michael Shulman. "Internal Parametricity, without an Interval". In: *Proceedings of the ACM on Programming Languages* 8.POPL (Jan. 2024). DOI: 10.1145/3632920

Homotopy type theory and cubical type theory

- Nicolai Kraus and Christian Sattler. "Higher Homotopies in a Hierarchy of Univalent Universes". In: *ACM Transactions on Computational Logic* 16.2 (Apr. 2015), 18:1–18:12. ISSN: 1529-3785. DOI: 10.1145/2729979

- Cyril Cohen, Thierry Coquand, Simon Huber, and Anders Mörtberg. "Cubical Type Theory: A Constructive Interpretation of the Univalence Axiom". In: *21st International Conference on Types for Proofs and Programs (TYPES 2015)*. Ed. by Tarmo Uustalu. Vol. 69. Leibniz International Proceedings in Informatics (LIPIcs). Dagstuhl, Germany: Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, 2018, 5:1–5:34. ISBN: 978-3-95977-030-9. DOI: 10.4230/LIPIcs.TYPES.2015.5

- Carlo Angiuli, Guillaume Brunerie, Thierry Coquand, Robert Harper, Kuen-Bang Hou (Favonia), and Daniel R. Licata. "Syntax and models of Cartesian cubical type theory". In: *Mathematical Structures in Computer Science* 31.4 (2021). Special issue on Homotopy Type Theory and Univalent Foundations, pp. 424–468. DOI: 10.1017/S0960129521000347

- Thierry Coquand, Simon Huber, and Anders Mörtberg. "On Higher Inductive Types in Cubical Type Theory". In: *Proceedings of the 33rd Annual ACM/IEEE Symposium on Logic in Computer Science*. LICS 2018. New York, NY, USA: ACM, 2018, pp. 255–264. ISBN: 978-1-4503-5583-4. DOI: 10.1145/3209108.3209197

★ Krzysztof Kapulkin and Peter LeFanu Lumsdaine. "The simplicial model of Univalent Foundations (after Voevodsky)". In: *Journal of the European Mathematical Society* 23 (6 2021), pp. 2071–2126. DOI: 10.4171/JEMS/1050

★ Emily Riehl and Michael Shulman. "A type theory for synthetic ∞-categories". In: *Higher Structures* 1.1 (2017), pp. 147–224. URL: https://arxiv.org/abs/1705.07442

Modal type theories

- Daniel Gratzer, Evan Cavallo, G. A. Kavvos, Adrien Guatto, and Lars Birkedal. "Modalities and Parametric Adjoints". In: *ACM Transactions on Computational Logic* 23.3 (Apr. 2022). ISSN: 1529-3785. DOI: 10.1145/3514241

- Daniel Gratzer, G. A. Kavvos, Andreas Nuyts, and Lars Birkedal. "Multimodal Dependent Type Theory". In: *Logical Methods in Computer Science* 17.3 (July 2021). DOI: 10.46298/lmcs-17(3:11)2021

- Robert Atkey. "Syntax and Semantics of Quantitative Type Theory". In: *Proceedings of the 33rd Annual ACM/IEEE Symposium on Logic in Computer Science*. LICS '18. New York, NY, USA: ACM, 2018, pp. 56–65. ISBN: 9781450355834. DOI: 10.1145/3209108.3209189

- Benjamin Moon, Harley Eades III, and Dominic Orchard. "Graded Modal Dependent Type Theory". In: *Programming Languages and Systems*. Ed. by Nobuko Yoshida. Cham: Springer International Publishing, 2021, pp. 462–490. ISBN: 978-3-030-72019-3

- Pritam Choudhury, Harley Eades III, Richard A. Eisenberg, and Stephanie Weirich. "A graded dependent type system with a usage-aware semantics". In: *Proceedings of the ACM on Programming Languages* 5.POPL (Jan. 2021). DOI: 10.1145/3434331

Type-checking and metatheory

- Thierry Coquand. "An algorithm for type-checking dependent types". In: *Science of Computer Programming* 26.1 (1996), pp. 167–177. ISSN: 0167-6423. DOI: https://doi.org/10.1016/0167-6423(95)00021-6

- Andreas Abel, Thierry Coquand, and Peter Dybjer. "On the Algebraic Foundation of Proof Assistants for Intuitionistic Type Theory". In: *Functional and Logic Programming*. Ed. by Jacques Garrigue and Manuel V. Hermenegildo. Berlin, Heidelberg: Springer Berlin Heidelberg, 2008, pp. 3–13. ISBN: 978-3-540-78969-7
- Andreas Abel, Joakim Öhman, and Andrea Vezzosi. "Decidability of conversion for type theory in type theory". In: *Proceedings of the ACM on Programming Languages* 2.POPL (Dec. 2017), 23:1–23:29. DOI: 10.1145/3158111
- Andreas Abel, Thierry Coquand, and Miguel Pagano. "A Modular Type-checking algorithm for Type Theory with Singleton Types and Proof Irrelevance". In: *Logical Methods in Computer Science* 7.2 (May 2011). DOI: 10.2168/LMCS-7(2:4)2011
- ★ T. Altenkirch, P. Dybjer, M. Hofmann, and P. Scott. "Normalization by evaluation for typed lambda calculus with coproducts". In: *Proceedings of the 16th Annual IEEE Symposium on Logic in Computer Science (LICS 2001)*. 2001, pp. 303–310. DOI: 10.1109/LICS.2001.932506

Conservativity

- Théo Winterhalter, Matthieu Sozeau, and Nicolas Tabareau. "Eliminating reflection from type theory". In: *Proceedings of the 8th ACM SIGPLAN International Conference on Certified Programs and Proofs*. CPP 2019. New York, NY, USA: ACM, 2019, pp. 91–103. ISBN: 9781450362221. DOI: 10.1145/3293880.3294095
- ★ Chris Kapulkin and Yufeng Li. *Extensional concepts in intensional type theory, revisited*. Preprint. Oct. 2023. arXiv: 2310.05706 [math.LO]. URL: https://arxiv.org/abs/2310.05706

Extending definitional equality

- Guillaume Allais, Conor McBride, and Pierre Boutillier. "New equations for neutral terms: a sound and complete decision procedure, formalized". In: *Proceedings of the 2013 ACM SIGPLAN Workshop on Dependently-Typed Programming*. DTP '13. New York, NY, USA: ACM, 2013, pp. 13–24. ISBN: 9781450323840. DOI: 10.1145/2502409.2502411
- Gaëtan Gilbert, Jesper Cockx, Matthieu Sozeau, and Nicolas Tabareau. "Definitional proof-irrelevance without K". in: *Proceedings of the ACM on Programming Languages* 3.POPL (Jan. 2019). DOI: 10.1145/3290316
- Jesper Cockx, Nicolas Tabareau, and Théo Winterhalter. "The taming of the rew: a type theory with computational assumptions". In: *Proceedings of the ACM on Programming Languages* 5.POPL (Jan. 2021). DOI: 10.1145/3434341
- Vilhelm Sjöberg and Stephanie Weirich. "Programming up to Congruence". In: *Proceedings of the 42nd Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*. POPL 2015. New York, NY, USA: ACM, 2015, 369–382. ISBN: 9781450333009. DOI: 10.1145/2676726.2676974

Elaborating dependent pattern matching

- Healfdene Goguen, Conor McBride, and James McKinna. "Eliminating Dependent Pattern Matching". In: *Algebra, Meaning, and Computation: Essays dedicated to Joseph A. Goguen on the Occasion of His 65th Birthday*. Ed. by Kokichi Futatsugi, Jean-Pierre Jouannaud, and José Meseguer. Berlin, Heidelberg: Springer Berlin Heidelberg, 2006, pp. 521–540. ISBN: 978-3-540-35464-2. DOI: 10.1007/11780274_27

- Jesper Cockx, Dominique Devriese, and Frank Piessens. "Pattern Matching without K". in: *Proceedings of the 19th ACM SIGPLAN International Conference on Functional Programming*. ICFP '14. New York, NY, USA: ACM, 2014, pp. 257–268. ISBN: 978-1-4503-2873-9. DOI: 10.1145/2628136.2628139

Universes

- Marc Bezem, Thierry Coquand, Peter Dybjer, and Martín Escardó. "Type Theory with Explicit Universe Polymorphism". In: *28th International Conference on Types for Proofs and Programs (TYPES 2022)*. Ed. by Delia Kesner and Pierre-Marie Pédrot. Vol. 269. Leibniz International Proceedings in Informatics (LIPIcs). Dagstuhl, Germany: Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2023, 13:1–13:16. ISBN: 978-3-95977-285-3. DOI: 10.4230/LIPIcs.TYPES.2022.13

Inductive types

- Peter Dybjer. "Inductive families". In: *Formal Aspects of Computing* 6.4 (July 1994), pp. 440–465. ISSN: 0934-5043. DOI: 10.1007/BF01211308

- Peter Dybjer and Anton Setzer. "Induction-recursion and initial algebras". In: *Annals of Pure and Applied Logic* 124.1 (2003), pp. 1–47. ISSN: 0168-0072. DOI: https://doi.org/10.1016/S0168-0072(02)00096-9

- Marcelo P. Fiore, Andrew M. Pitts, and S. C. Steenkamp. "Quotients, inductive types, and quotient inductive types". In: *Logical Methods in Computer Science* 18.2 (June 2022). DOI: 10.46298/lmcs-18(2:15)2022